CEE 491/591 — Groundwater Hydrology
Assignment 3: Darcy's Law and Intrinsic Permeability

Arash Massoudieh

Module 3

Submission Instructions

All answers must be submitted as a single **Excel file**.

- Each problem must be solved on a **separate worksheet**.
- **All calculations must be performed using Excel formulas** (no hard-coded numerical results).
- All formulas must be **visible and preserved** in the submitted file.
- Clearly label the given data, intermediate calculations, and final answers, with units for all results.
- Write short explanations in a text cell next to the related calculation.
- Upload the `.xlsx` file to **Brightspace** → **Assignments**.

Problem 1: Hydraulic Conductivity and Darcy's Law (20 points)

A horizontal sand aquifer has a hydraulic conductivity of $K = 5$ m/day. Two piezometers installed 40 m apart measure hydraulic heads of 12.4 m and 11.6 m, respectively. The saturated thickness of the aquifer is 6 m, and the width perpendicular to flow is 10 m.

(a) Calculate the hydraulic gradient.

(b) Compute the horizontal specific discharge (Darcy flux).

(c) Calculate the volumetric flow rate through the aquifer cross-section.

(d) Briefly explain why the Darcy velocity differs from the average pore-water velocity.

Problem 2: Three-Dimensional Groundwater Flow (40 points)

Three observation wells (A, B, and C) are installed in an unconfined aquifer of fine to medium sand.

- Measured heads: $h_A = 128.45$ m, $h_B = 127.92$ m, $h_C = 128.78$ m.
- Well A is at the origin; well B is 500 m east of A; well C is 400 m north of A.

- Horizontal hydraulic conductivity $K_h = 15$ m/day; porosity $n = 0.28$; regional recharge $R = 600$ mm/yr.
- Assume steady state, a homogeneous aquifer near the wells, and a vertical specific discharge equal to the recharge rate. The average vertical hydraulic gradient is $\partial h/\partial z = 0.008$.

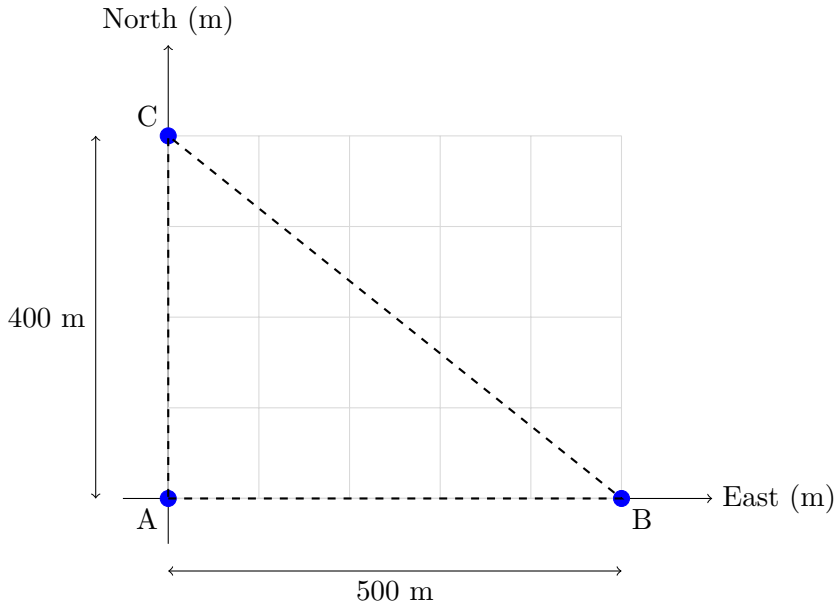


Figure 1: Plan view of observation well locations

1. **Hydraulic gradient.** Approximate the head surface as $h(x, y) = a + bx + cy$ and solve for the coefficients from the three heads. Report (a) $\partial h/\partial x$, (b) $\partial h/\partial y$, (c) the magnitude of the horizontal gradient, and (d) the direction of horizontal flow in degrees from east.
2. **Horizontal Darcy flux.** With $q_x = -K_h\,\partial h/\partial x$ and $q_y = -K_h\,\partial h/\partial y$, report (a) q_x, (b) q_y, and (c) $q_h = \sqrt{q_x^2 + q_y^2}$, in m/day.
3. **Vertical hydraulic conductivity.** Report (a) the recharge rate in m/day, (b) K_v from $q_z = K_v\,\partial h/\partial z$, (c) the anisotropy ratio K_h/K_v, and (d) whether this ratio is reasonable for a sand aquifer (one sentence).
4. **Total specific discharge.** Report (a) $|\vec{q}| = \sqrt{q_x^2 + q_y^2 + q_z^2}$ and (b) the vertical component as a percentage of $|\vec{q}|$.
5. **Pore-water velocity.** Report (a) the horizontal and (b) the vertical pore-water velocities, $v = q/n$, and (c) explain in one sentence why the pore-water velocity differs from the Darcy flux.

Problem 3: Intrinsic Permeability and DNAPL Migration (40 points)

Hydraulic conductivity depends on both the porous medium and the fluid: $K = k\,\rho g/\mu$, where k is the intrinsic permeability (m^2), a property of the medium alone. Unit conversion: 1 darcy $= 9.87 \times 10^{-13}$ m^2; $g = 9.81$ m/s^2.

Fluid	ρ (g/cm^3)	μ (N·s/m^2)	T (°C)
Water	1.000	1.00×10^{-3}	20
Water	0.998	8.90×10^{-4}	25
PCE (tetrachloroethylene)	1.63	9.0×10^{-4}	20

An underground storage tank has leaked PCE (tetrachloroethylene, a dense non-aqueous phase liquid, DNAPL) into a sandy aquifer. Aquifer testing with water at 20 °C gives $K_w = 4.5 \times 10^{-4}$ m/s. The porosity is $n = 0.32$, the hydraulic gradient driving PCE flow is $i = 0.0015$, and the summer groundwater temperature rises to 25 °C.

A. **Intrinsic permeability (8 pts):** compute k in (a) m^2 and (b) darcy.

B. **PCE hydraulic conductivity (6 pts):** compute K_{PCE} at 20 °C.

C. **PCE transport velocity (8 pts):** compute (a) the Darcy flux, (b) the pore velocity, and (c) the distance travelled in one year.

D. **Temperature effect on water (8 pts):** compute (a) K for water at 25 °C and (b) the percentage change from 20 to 25 °C.

E. **Conductivity ratio (6 pts):** compute $K_{PCE}/K_{w,20}$. Does PCE flow faster or slower than water in this aquifer, and why?

F. **Discussion (4 pts + 2 bonus):** a monitoring well is 50 m downgradient of the leak. Estimate the time for PCE to reach it, travelling at the pore velocity from Part C. *Bonus:* name two factors not included here that could make the actual arrival time differ.